

Warm-up!

Let θ be an angle graphed in standard form. If the terminal ray of θ passes through the point $(-2, 5)$, find...

$\sin \theta$

$\cos \theta$

$\tan \theta$

Trig Functions, Day 4

- Speed Quiz: Q1+Quadrants
- (3.5) Even/Odd Properties
- (3.10) Solving Basic Trig Equations

Homework: Solving Basic Trig Equations + More Evaluating

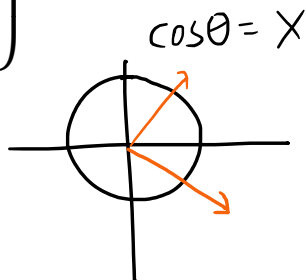
Quiz Next Class

- Vocab: Reference Angle, Coterminal Angle, Standard Form, Terminal Ray
- Points on Circles in Planes
- Evaluating Trig Functions of Obtuse and Negative Angles

Even-Odd Identities:

$$\left. \begin{array}{l} \cos(-\theta) = \cos \theta \\ \sec(-\theta) = \sec \theta \end{array} \right\} \text{Even Functions}$$

$$\left. \begin{array}{l} \sin(-\theta) = -\sin \theta \\ \csc(-\theta) = -\csc \theta \\ \tan(-\theta) = -\tan \theta \\ \cot(-\theta) = -\cot \theta \end{array} \right\} \text{Odd Functions}$$



$$\begin{aligned} \sin\left(-\frac{\pi}{6}\right) &= -\sin \frac{\pi}{6} \\ &= -\frac{1}{2} \end{aligned}$$

$$\cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$$

You don't need to do these on your own. Let's just do them together, writing a ton of equivalent expressions. Your job is to make sure you understand how each step is done/why things are equal.

$$\begin{aligned}
 \sin\left(-\frac{7\pi}{6}\right) &= -\sin\left(\frac{7\pi}{6}\right) = -\sin\left(-\frac{5\pi}{6}\right) \\
 \sin\left(-\frac{7\pi}{6}\right) &= +\sin\left(\frac{\pi}{6}\right) = -\left(-\sin\left(\frac{5\pi}{6}\right)\right) \\
 &= \frac{1}{2} \qquad \qquad \qquad = \sin\left(\frac{5\pi}{6}\right) \\
 \cos\left(-\frac{5\pi}{2}\right) &=
 \end{aligned}$$

$$\begin{aligned}
 \csc\left(\frac{23\pi}{6}\right) &= \csc\left(\frac{11\pi}{6}\right) = \csc\left(-\frac{\pi}{6}\right) = -\csc\left(\frac{\pi}{6}\right) \\
 &= -2
 \end{aligned}$$

$$\frac{23\pi}{6} - \frac{12\pi}{6} = \frac{11\pi}{6}$$

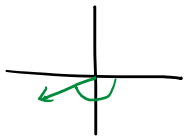
If $\sin(-t) = \frac{3}{8}$, find...

$$\sin(t) = -\frac{3}{8}$$

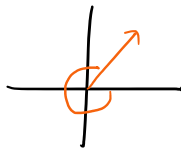
$$\csc(t) = -\frac{8}{3}$$

Evaluate

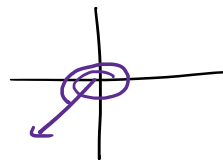
$$\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$



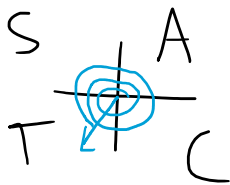
$$\tan\left(-\frac{5\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$



$$\sec\left(\frac{13\pi}{4}\right) = -\sec\left(\frac{\pi}{4}\right) = -\sqrt{2}$$



$$\cos\left(\frac{16\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$



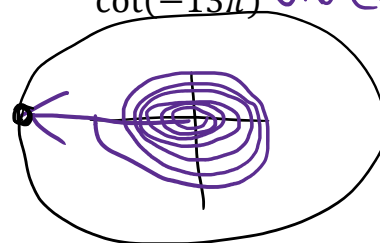
$$\csc\left(-\frac{3\pi}{2}\right) = 1$$



$$\sin\theta = y$$

$$\csc\theta = \frac{1}{y}$$

$$\cot(-13\pi) \text{ undefined}$$



$$\tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y} = -\frac{1}{0}$$

$$\frac{16\pi}{3} - \frac{6\pi}{3} - \frac{6\pi}{3} = \frac{4\pi}{3}$$

Which is not equivalent to the others

$$\sin\left(\frac{7\pi}{6}\right)$$

$$\tan\left(-\frac{4\pi}{3}\right)$$

$$\cos\left(\frac{2\pi}{3}\right)$$

$$\cot\left(\frac{11\pi}{6}\right)$$

$$\sin\left(-\frac{11\pi}{6}\right)$$

$$\tan\left(-\frac{\pi}{3}\right)$$

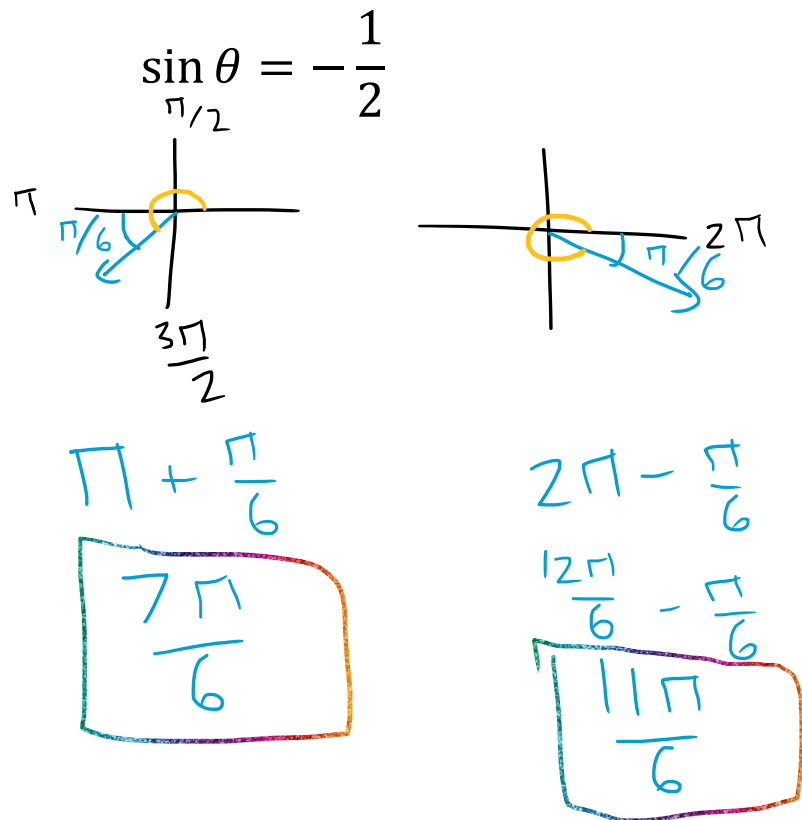
$$\cos\left(-\frac{2\pi}{3}\right)$$

$$\cot\left(-\frac{5\pi}{6}\right)$$

Solving Basic Trig Equations

- 1) Isolate the trigonometric expression
- 2) Figure out the possible quadrants based on the sign of the number (i.e. only pay attention to positive vs negative)
- 3) Figure out the reference angle based on the magnitude of the number (i.e. ignore negative)
- 4) Combine info to determine θ

Example: Find the two values of θ in the interval $0 \leq \theta < 2\pi$ that solve the equation.



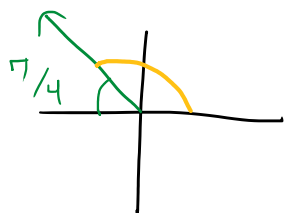
Find the two values of θ in the interval
equation.

that solve the

$$\sqrt{2} \boxed{}$$

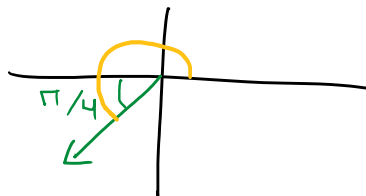
$$\cos \theta = -\frac{1}{\sqrt{2}}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$



$$\pi - \frac{\pi}{4}$$

$$\theta = \frac{3\pi}{4}$$

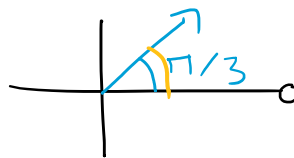


$$\pi + \frac{\pi}{4}$$

$$\theta = \frac{5\pi}{4}$$

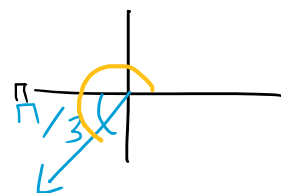
$$\cot(\theta) = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\tan(\theta) = \sqrt{3}$$



$$\pi/3$$

$$\theta = \frac{\pi}{3}$$



$$\pi + \frac{\pi}{3}$$

$$\theta = \frac{4\pi}{3}$$

Find all values of θ in the interval $0 \leq \theta < 2\pi$ that solve the equation.

$$4(\cos \theta)^2 = 1$$
$$\sqrt{(\cos \theta)^2} = \sqrt{\frac{1}{4}}$$
$$\cos \theta = \frac{1}{2} \qquad \cos \theta = -\frac{1}{2}$$


Diagram showing the unit circle for $\cos \theta = \frac{1}{2}$. The angle $\theta = \pi/3$ is marked in the first quadrant.

$$\pi/3$$


Diagram showing the unit circle for $\cos \theta = -\frac{1}{2}$. The angles $2\pi - \pi/3$ and $5\pi/3$ are marked in the fourth and third quadrants, respectively.

$$2\pi - \frac{\pi}{3}$$
$$\frac{5\pi}{3}$$


Diagram showing the unit circle for $\cos \theta = -\frac{1}{2}$. The angles $2\pi - \pi/3$ and $5\pi/3$ are marked in the fourth and third quadrants, respectively.

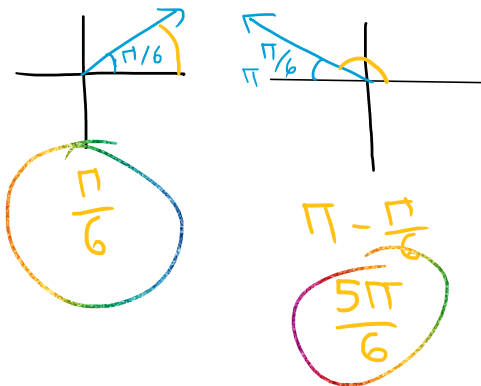
Find all solutions in the interval $0 \leq \theta < 2\pi$

$$(2 \sin \theta - 1)(\tan \theta + 1) = 0$$

$$2 \sin \theta - 1 = 0$$

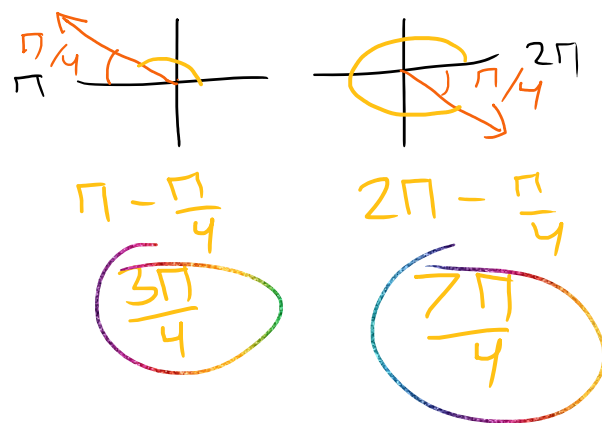
$$2 \sin \theta = 1$$

$$\sin \theta = \frac{1}{2}$$



$$\tan \theta + 1 = 0$$

$$\tan \theta = -1$$



Solve over $[0, 2\pi)$

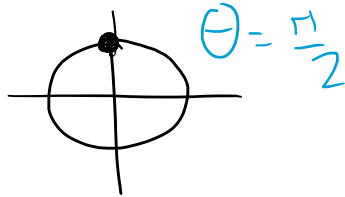
What makes this question different?

$$\sin(\theta) = -1$$

$$(\sin \theta - 1)(\sin \theta + 2) = 0$$

$$\sin \theta - 1 = 0$$

$$\sin \theta = 1$$



$$\sin \theta + 2 = 0$$

$$\sin \theta = -2$$

impossible

